

Optical Zeno Gate: Bounds for Fault Tolerant Operation

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In principle the Zeno effect controlled-sign gate of Franson et al's (PRA 70, 062302, 2004) is a deterministic two-qubit optical gate. However, when realistic values of photon loss are considered its fidelity is significantly reduced. Here we consider the use of measurement based quantum processing techniques to enhance the operation of the Zeno gate. With the help of quantum teleportation, we show that it is possible to achieve a Zeno CNOT gate (GC-Zeno gate) that gives (near) unit fidelity and moderate probability of success of 0.76 with a one-photon to two-photon transmission ratio $\kappa = 10^4$. We include some mode-mismatch effects and estimate the bounds on the mode overlap and κ for which fault tolerant operation would be possible.

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I. INTRODUCTION

Photons are a natural choice for making qubits because the quantum information encoded can have a long decoherence time and is easy to manipulate and measure. Also, photonic qubits are the only type of qubits that are feasible for long distance quantum communication. Quantum information processing requires universal two-qubit entangling gates. Knill et al [1] showed that it is theoretically possible to do scalable quantum computing with linear optics by using measurement induced interactions to perform the two-qubit gates. However, despite a continuous effort in reducing the resource requirement [2, 3, 4, 5], the resource overhead is still high for linear optical quantum computing. Franson et al[6] proposed the use of two-photon absorption non-linearity and exploiting the quantum Zeno effect to implement a control sign (CZ) gate that requires much less resources than linear optics schemes. However, the problem with the Zeno gate is that photon loss affects the performance of the Zeno gate significantly and the single photon to two-photon loss ratio requirement is very stringent. One solution could be to combine measurement induced quantum processing with the Zeno gate. Previously we have shown that when using the Zeno gate for qubit fusion [8], state distillation [9] with post-selection can boost the gate fidelity to unity and that for less stringent absorption ratios the gate has an advantage in success probability over linear optics gates for fusing clusters of qubits. These clusters of qubits can then be used for cluster state quantum computing [10]. In addition, Myers and Gilchrist [7] have shown that the performance of the Zeno gate may be enhanced by using error correction codes such as redundancy and parity encoding.

Here we design a high fidelity Zeno CNOT gate suitable for circuit-based quantum computing. Although with the current estimate of the photon loss ratio, only a poor fidelity Zeno gate is directly achievable, here we show that it is possible to use two of these Zeno gates

to do Bell measurements and implement a Gottesman-Chuang[11] teleportation type of CNOT gate (GC-Zeno gate) that, like the fusion gate, gives high fidelity via state distillation and moderate success probability via partially off-line state preparation. We include the effect of mode-mismatch and detector efficiency on the scheme and estimate lower bounds on the parameter which in principle allow fault tolerant operation.

The paper is arranged in the following way. The introduction continues with a subsection on the Zeno CZ gate, which describes the scheme and modelling of the gate and give descriptions on the modelling parameters that are also used for modelling the GC-Zeno CZ gate. In section II, we discuss the GC-Zeno gate and the effect of mode-mismatch and detector efficiency on the gate. In section III, we give estimates of the lower bounds on the photon loss ratio and mode-matching, followed by a subsection on the advantage in using state distillation. We conclude and summarize in section IV.

Ia. Zeno CZ Gate

Franson et al's control sign gate scheme consists of a pair of optical fibers weakly evanescently coupled and doped with two-photon absorbing atoms. The purpose of the two-photon absorbers is to suppress the occurrence of two photon state components in the two fibre modes via the Quantum Zeno effect. This allows the state to remain in the computational basis. After a length of fibre corresponding to a complete swap of the two fibre modes, a π phase difference is produced between the $|11\rangle$ term and the other three basis terms. After swapping the fibre modes by simply crossing them, a CZ gate is achieved. The gate becomes near deterministic and performs a near unitary operation when the Quantum Zeno effect is strong and photon loss is insignificant. However, with current technology, the strength of the

Quantum Zeno effect is a few orders of magnitude below what is required, and thus the Zeno gate has significant photon loss.

In [8], the gate is modelled as a succession of n weak beamsplitters followed by two-photon absorbers as shown in Fig. 1. As $n \rightarrow \infty$ the model tends to the continuous coupling limit envisaged for the physical realization. The gate operates on the single-rail encoding for which $|0\rangle_L = |0\rangle$ and $|1\rangle_L = |1\rangle$ with the kets representing photon Fock states. Fig. 2 shows how the single rail CZ can be converted into a dual rail [13] CZ with logical encoding $|0\rangle_L = |H\rangle$ and $|1\rangle_L = |V\rangle$. Let L be the total length of n absorbers. Also, let $\gamma_1 = \exp(\frac{-\lambda}{n\kappa})$ and $\gamma_2 = \exp(\frac{-\lambda}{n})$ be the probability of single photon and two-photon transmission respectively for one absorber. Here the parameter $\lambda = \chi L$, where L is the length of the absorber and χ is the corresponding proportionality constant related to the absorption cross section. Furthermore, κ specifies the relative strength of the two transmissions and relates them by $\gamma_2 = \gamma_1^\kappa$. This CZ gate does the following operation:

$$\begin{aligned} |00\rangle &\rightarrow |00\rangle \\ |01\rangle &\rightarrow \gamma_1^{n/2} |01\rangle \\ |10\rangle &\rightarrow \gamma_1^{n/2} |10\rangle \\ |11\rangle &\rightarrow -\gamma_1^n \tau |11\rangle + f(|02\rangle, |20\rangle) \end{aligned} \quad (1)$$

where the new expression for τ is given by:

$$\begin{aligned} \tau_{n,\lambda} &= \frac{2^{-\frac{3}{2}-n}}{d_{n,\lambda}} \left((g_{n,\lambda} + \frac{d_{n,\lambda}}{\sqrt{2}})^n (\sqrt{2}d_{n,\lambda} - h_{n,\lambda}) \right. \\ &\quad \left. + (g_{n,\lambda} - \frac{d_{n,\lambda}}{\sqrt{2}})^n (\sqrt{2}d_{n,\lambda} + h_{n,\lambda}) \right) \\ d_{n,\lambda} &= \sqrt{(1 + \cos \frac{2\pi}{n})(1 + \gamma_2) + 2\sqrt{\gamma_2}(\cos(\frac{2\pi}{n}) - 3)} \\ g_{n,\lambda} &= (\cos \frac{\pi}{n})(\sqrt{\gamma_2} + 1) \\ h_{n,\lambda} &= 2(\cos \frac{\pi}{n})(\sqrt{\gamma_2} - 1) \end{aligned} \quad (2)$$

The explicit form of the $|02\rangle, |20\rangle$ state components are suppressed, as they lie outside the computational basis and so do not explicitly contribute to the fidelity.

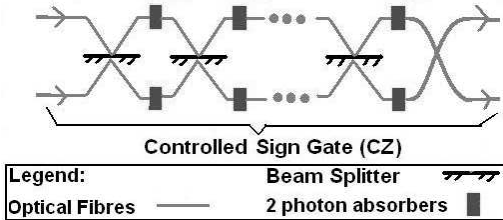


FIG. 1: Construction of our CZ gate.

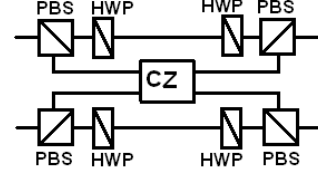


FIG. 2: CZ gate in dual rail implementation.

From equation 1, it is clear that the amplitude of the four computational states are unequal and this lowers the gate fidelity. With the current best estimate of $\kappa = 10^4$, the unherald fidelity is only 0.94. If the gate is used in a measurement based strategy then state distillation can be used and the fidelity of the gate can be improved by trading off some success probability. Figure 3 shows the distilled Zeno CZ gate circuit. The τ gate is simply an interferometer consisting of two 50-50 beam splitters with a two-photon absorber in each arm, which gives operation: $|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow \sqrt{\gamma_1'} |01\rangle$, $|10\rangle \rightarrow \sqrt{\gamma_1'} |10\rangle$, and $|11\rangle \rightarrow \gamma_1' \tau |11\rangle$. Here $\gamma_1' = \tau^{1/\kappa}$ is the single photon transmission coefficient of the absorber. The distillation beam splitters labelled 1 to 3 have transmission coefficient $\sqrt{\gamma_1'}$, $\sqrt{\gamma_1'}$, $\sqrt{\gamma_1' \gamma_1' \tau}$ respectively. With these distillations in place, the operation of the distilled Zeno CZ gate is:

$$\begin{aligned} |00\rangle &\rightarrow \gamma_1^n \gamma_1' \tau |00\rangle \\ |01\rangle &\rightarrow \gamma_1^n \gamma_1' \tau |01\rangle \\ |10\rangle &\rightarrow \gamma_1^n \gamma_1' \tau |10\rangle \\ |11\rangle &\rightarrow -\gamma_1^n \gamma_1' \tau |11\rangle + f(|02\rangle, |20\rangle) \end{aligned} \quad (3)$$

After measuring the output (detectors at output are not shown in fig.3) and treating the photon bunching terms ($|02\rangle, |20\rangle$) and the terms with photon loss as failures (excluded from Eq. 3 for clarity), renormalising the states gives unit heralded fidelity independent of λ and probability of success $P_s = \gamma_1^{2n} \gamma_1'^2 \tau^2 = e^{-2\lambda/\kappa} \tau^{2+2/\kappa}$.

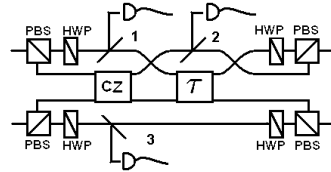


FIG. 3: Schematic of distilled Zeno CZ gate.

II. ZENO GATE USING GOTTESMAN-CHUANG SCHEME

As argued above, state distillation can improve the fidelity of the Zeno gate to unity by trading off success probability. However, the output of the distilled Zeno

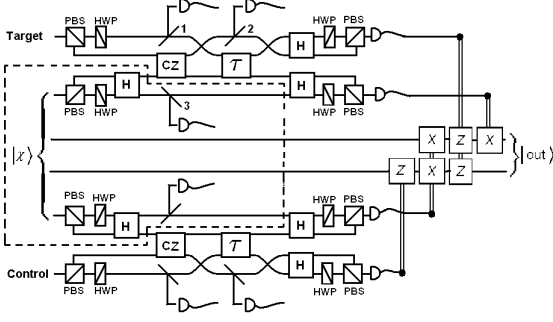


FIG. 4: Schematic of GC-Zeno gate. The state $|\chi\rangle$ is $((|00\rangle + |11\rangle)|00\rangle + (|01\rangle + |10\rangle)|11\rangle)/2$

gate contains terms outside the computational basis due to photon loss and photon bunching. Hence if we want the gate to have unit fidelity, it is necessary to measure the output and exclude these failure terms by post-selection. However, such post-selection means that the gate can no longer be used directly as a CNOT gate for circuit-based quantum computing.

Gottesman and Chuang [11] showed the viability in using state teleportation and single qubit operations to implement a CNOT gate. The scheme requires the four qubit entangled state $|\chi\rangle = ((|00\rangle + |11\rangle)|00\rangle + (|01\rangle + |10\rangle)|11\rangle)/2$. Preparing the entangled state requires a CZ operation, which can be done off-line with linear optics with high fidelity. Bell measurements are made between the input qubits and the first and last qubits of $|\chi\rangle$. The measurement results are fed forward for some single qubit corrections such that the circuit gives a proper CNOT operation. Here we propose using such scheme, as shown in figure 4, to implement a GC-Zeno CNOT gate with high fidelity. Since this gate includes state distillation, post-selection and off-line state preparation, the gate has unit fidelity (under perfect mode-matching) and moderate success probability. Figure 5 plots the probability of success against the one-photon to two-photon transmission ratio κ . It shows that with $\kappa = 10^4$ (current best estimate), the probability of success is about 0.76, which is better than the break even point of 0.25 for the linear optics version of this gate [1].

IIa. Effect of Mode-Mismatch

From source preparation to gate operation to detection, mode-mismatch is an unavoidable issue in optical quantum computing that causes unlocated errors which lowers the fidelity of the device[15]. Fortunately, with the help of quantum error correction, a certain amount of unlocated error rate, including but not limited to mode-mismatch errors, can be tolerated. A reliable quantum

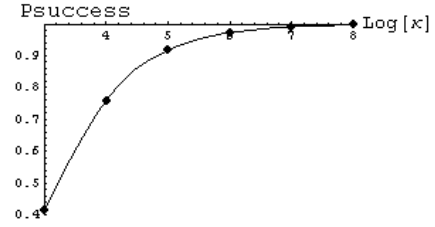


FIG. 5: Plot of probability of success versus $\log(\kappa)$ (in base 10) for GC-Zeno gate. Note that the success probability is not one at $\kappa = 10^8$, but that the curve asymptotically approaches one for very large κ . Result is per two input qubits. Detector inefficiency is taken into account in accord with Dawson et al's bound.

gate must therefore have unlocated error rates below this threshold.

The dominant source of mode-mismatch error in the GC-Zeno gate is from the CZ gate and τ gate, where two-photon interaction occurs. Here we follow Rohde et al's [14] analysis to examine the effect of such error. We take the simplest model in which the mode-mismatch is present between the photons entering the gate but remain constant through the gate. In this case, the mode-mismatch in two-photon interaction can be analysed as having two-photons fail to interact with some probability. This allows us to write the operations for the CZ gate as follow, where $0 < \Gamma < 1$ quantifies the overlap of the two wavepackets. Γ^2 is the probability that the two photons successfully interacted and $\Gamma = 0$ for completely mode-mismatched and $\Gamma = 1$ for completely mode-matched. The bar in the $|\bar{1}1\rangle$ term indicates mode-mismatched component of the state.

$$\begin{aligned}
 |00\rangle &\rightarrow |00\rangle \\
 |01\rangle &\rightarrow \sqrt{\gamma_1^n} |01\rangle \\
 |10\rangle &\rightarrow \sqrt{\gamma_1^n} |10\rangle \\
 |11\rangle &\rightarrow -\Gamma \gamma_1^n \tau |11\rangle + \sqrt{1 - \Gamma^2} \gamma_1^n |\bar{1}1\rangle \\
 &\quad + f(|02\rangle, |20\rangle)
 \end{aligned} \tag{4}$$

And similarly for the operations of τ gate:

$$\begin{aligned}
 |00\rangle &\rightarrow |00\rangle \\
 |01\rangle &\rightarrow \sqrt{\gamma_1'} |01\rangle \\
 |10\rangle &\rightarrow \sqrt{\gamma_1'} |10\rangle \\
 |11\rangle &\rightarrow \Gamma \gamma_1' \tau |11\rangle + \sqrt{1 - \Gamma^2} \gamma_1' |\bar{1}1\rangle \\
 &\quad + f(|02\rangle, |20\rangle)
 \end{aligned} \tag{5}$$

With the equations for the CZ and τ gate[16], and given a normalized input state $(\alpha|00\rangle + \beta|01\rangle + \delta|10\rangle + \epsilon|\bar{1}1\rangle)$, we can obtain analytical expression for the fidelity F (per qubit) and success probability P_s (per qubit) of

the GC-Zeno gate as follow. Equation 6 and 7 show that both the fidelity and success probability are state dependent due to mode-mismatch. The worst case of fidelity occurs when the input state is the equal superposition state $(|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2$ (i.e. $\alpha = \delta = \beta = \epsilon = 1/2$) and the worst case of success probability occurs when the input state is the pure state $|11\rangle$ (i.e. $\alpha = \beta = \delta = 0$ and $\epsilon = 1$).

$$F = \frac{\alpha^* A_1 + \beta^* A_2 + \delta^* A_3 + \epsilon^* A_4}{\sqrt{|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2}} \quad (6)$$

$$P_s = \frac{e^{-2\lambda/\kappa} \tau^{2/\kappa}}{2(1 + e^{-\lambda/\kappa} \tau^{2+1/\kappa})} \times \sqrt{|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2} \quad (7)$$

where $a_1 = (\tau + \tau\Gamma + \sqrt{1 - \Gamma^2})$, $a_2 = (\tau - \tau\Gamma + \sqrt{1 - \Gamma^2})$, $a_3 = (\tau - \tau\Gamma - \sqrt{1 - \Gamma^2})$, $a_4 = (\tau + \tau\Gamma - \sqrt{1 - \Gamma^2})$, and $A_1 = \alpha a_1^2 + \beta a_1 a_2 + \delta a_1 a_2 + \epsilon a_2^2$, $A_2 = \alpha a_1 a_3 + \beta a_2 a_3 + \delta a_1 a_4 + \epsilon a_2 a_4$, $A_3 = \alpha a_1 a_3 + \beta a_1 a_4 + \delta a_2 a_3 + \epsilon a_2 a_4$, $A_4 = \alpha a_3^2 + \beta a_3 a_4 + \delta a_3 a_4 + \epsilon a_4^2$

IIb. Effect of Detector Efficiency

In practice, even for the most advanced photon detector, detector inefficiency is always present. The effect of this noise is to reduce the probability of success of the gate but not the fidelity because the errors are locatable.

III. ESTIMATE OF BOUNDS FOR FAULT TOLERANCE

We now wish to estimate lower bounds on the mode-matching, Γ , and photon loss ratio, κ , that will still allow fault-tolerant operation. We allow a small amount of detector inefficiency but assume all other parameters are ideal. To make this estimate we directly use the bounds obtained by Dawson et al [12] for a deterministic error correction protocol. For this protocol, they numerically derived one bound using the 7-qubit Steane code and another bound using the 23-qubit Golay code.

In order to use the Dawson et al's bounds we need to identify the unlocated and located error rates for our gate. In general, the unlocated error rate is less than $1 - F$ but here we take it to be $1 - F$ because in our analysis, γ is almost 1, which means the other terms involved are very small. The located error rate is simply $1 - P_s$ (both F and P_s are per qubit). Using these relationships, we convert each of the bounds into a fidelity versus success probability bound. For a gate built with two-photon absorbers that have a certain single-photon to two-photon transmission ratio κ , we can find an optimal λ (i.e. choosing an optimal absorber length) that

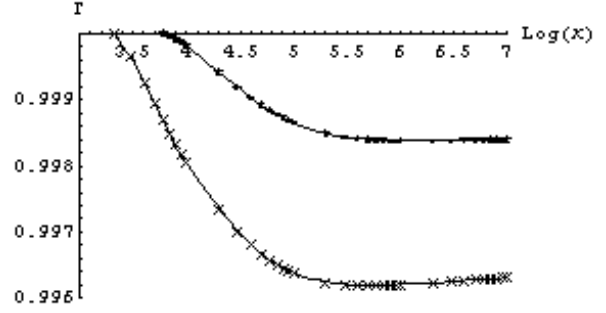


FIG. 6: Lower bounds of amount of mode-matching Γ required for a fault tolerant GC-Zeno gate versus single-photon to two-photon transmission ratio κ . The bounds are derived from Dawson et al's [12] results on deterministic error correction protocol. The top and bottom curves are for the 7-qubit Steane code and the 23-qubit Golay code respectively. Above the curves are the regions where the amount of mode-mismatch is tolerable. Here we have used the worst case input.

gives a maximum success probability. Hence, by matching the success probability with the bound, we can determine the corresponding fidelity threshold and therefore find the least amount of mode-matching required for fault tolerant gate operation. We note that the error model used by Dawson et al is specific to their optical cluster state architecture and will differ in detail from the appropriate error model for the GC-Zeno gate. Nonetheless we assume that a comparison based on the total error rates will give a good estimate of the bounds.

Figure 6 shows the lower bounds on the mode-matching parameter Γ for a gate with a certain κ . Since the fidelity and success probability are state dependent due to mode-mismatch, in that figure, we have plotted for the case of worst fidelity input state (i.e. the equal superposition state). The top and bottom curves are best fit curves for using the 7-qubit Steane code and the 23-qubit Golay code respectively. The curves show that highly mode-matched photons are essential for robust gate operation. With the worst fidelity input state, $(|00\rangle + |01\rangle + |10\rangle + |11\rangle)/2$, for the Steane code, the lowest Γ required for fault tolerant operation is about 0.998, where $\kappa = 10^6$, and for the Golay code, the lowest Γ required is about 0.996, where $\kappa = 5 \times 10^5$. With the worst success probability input state, $|11\rangle$, for the Steane code, the lowest Γ required for fault tolerant operation is about 0.995, where $\kappa = 10^6$, and for the Golay code, the lowest Γ required is about 0.989, where $\kappa = 5 \times 10^5$. Figure 6 also shows that under (near) perfect mode-matching, the required κ can be as low as approximately 6000 for the Steane code and 2000 for the Golay code. Two-photon absorbers with such κ values may be achievable with the best of current technology.

IIIa. Advantage of Using State Distillation

State distillation allows us to trade off some success probability against fidelity for the GC-Zeno gate, or in other words, reducing the unlocated error rate by having a larger located error rate. Since the deterministic error correction protocol can tolerate both unlocated and located errors, therefore we should ask whether state distillation is truly advantageous? We can answer this question by comparing two GC-Zeno gates in the case of perfect mode matching, where one has complete distillation and the other has no distillation. For the case of complete distillation, the deterministic error correction protocol with the 7-qubit Steane code can tolerate errors of a GC-Zeno gate with $\kappa = 6100$, and with the 23-qubit Golay code, it can tolerate errors of a GC-Zeno gate with $\kappa = 2100$. For state distillation to be advantageous under the same protocol, these values of κ must be smaller than the values of κ for the case of no distillation[17].

For the case of no distillation, the fidelity and success probability of the gate becomes state dependent. In the parameters space of interest, the input that gives the worst fidelity is $(|00\rangle + |01\rangle + |10\rangle - |11\rangle)/2$. With this input state, we find that for the protocol using the 7-qubit Steane code and no distillation, the critical κ is 12000. Similarly, for the protocol using the 23-qubit Golay code and no distillation, the critical κ is 4300. With an arbitrary amount of distillation, the value of κ lies between the limit of no distillation and full distillation cases. Hence it is evident that state distillation is advantageous. Also, it should be noted it is better to have only located error, which is the case when there is full distillation, than have both located and unlocated errors, which is the case when no or some distillation is utilized.

IV. CONCLUSION

In this paper, we have shown that it is possible to build a high fidelity Zeno CNOT gate with two distilled Zeno gates implemented in the Gottesman-Chuang teleportation CNOT scheme. For one-photon to two-photon transmission ratio $\kappa = 10^4$ (current best estimate), the gate has a success probability of 0.76 under perfect mode-matching. When including measurement noise that equals one-tenth of the gate's noise, and the effect of mode-mismatch in the CZ and τ gate, we find

that with the deterministic error correction protocol using the 7-qubit Steane code, the lowest Γ required for fault tolerant gate operation is 0.998, where $\kappa = 10^6$. For using the 23-qubit Golay code, the lowest Γ required is 0.996, where $\kappa = 5 \times 10^5$. Hence, the requirement on mode-matching is stringent for a fault tolerant GC-Zeno gate.

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- [15] Note that mode mismatch in CZ and τ gate causes unlocated error that lowers the fidelity, in which we find that it cannot be improved with state distillation.
- [16] Due to mode-mismatch, the τ gate is less effective in two-photon distillation. It is true that we can increase the two-photon absorption strength in the τ gate to make up for the inefficiency. However, here we assume that we do not know the mode-matching parameter Γ of the input wavepackets, and therefore this adjustment cannot be made. In addition, increasing the two-photon distillation will increase single-photon loss as well, which lowers the probability of success.
- [17] An ideal Zeno gate requires strong quantum Zeno effect and strong quantum Zeno effect corresponds to a large κ value. However such large non-linearity is difficult to engineer. Hence it is desirable to have a gate that works with modest κ .